School of Computer Science

60-265-01 Computer Architecture and Digital Design

Winter 2009 Semester

Midterm Examination # 1
Wednesday, February 25, 2009

Student Name: ________________ ___________________

First Name Family Name

Student ID Number: _____________________

Duration of examination: 75 minutes

1. Answer all questions on this examination paper in the space provided.
2. This is a closed-book examination – no notes or books or electronic computing or storage devices may be used.
3. Do not copy from other students or communicate in any way. All questions will be answered only by the attending proctors.
4. All students must remain seated during the last 5 minutes of the examination.
5. The examination must be surrendered immediately when the instructor announces the end of the test period.
6. Each student <u>must</u> sign the examination list before leaving the classroom.

Total mark obtained: ______________________

Maximum mark: 44

Question 1. [5 marks]
Answer each multiple choice question below by **circling or underlining** the single, most correct choice of answer.

A. The word *bit* is derived from _________ .

a. a tiny piece of a computer memory
b. a positional value
c. a binary digit
d. none of these

B. The word *digital* implies that the information in the computer is represented by variables that ___________ .

a. Take an arbitrary number of values
b. Take an arbitrary number of discrete values
c. Take a limited number of values
d. Take a limited number of discrete values

C. Computer ___________ is concerned with the way the hardware components operate and the way they are connected together to form the computer system.

a. Organization
b. Design
c. Architecture
d. Networking

D. Computer ___________ is concerned with the structure and behavior of the computer as seen by the user.

a. Organization
b. Design
c. Architecture
d. Networking

E. The combination of a limited number of fundamental gates to form a functional circuit is called __________ .

a. Boolean logic
b. Small-scale integration
c. Medium-scale integration
d. Large-scale integration

Question 2. [6 marks]
Using Boolean algebra, prove the following theorems. For each step in the proofs state the specific axiom that is used. Each part is worth 3 marks. [NOTE: The full list of all axioms is provided for each student on the last page of this examination.]

(a) $X.X = X$

Proof:

X.X	= (X.X)+0	Identity
	= (X.X) + (X.X′)	Complementation
	= X.(X+X′)	Distributivity
	= X.1	Complementation
	= X	Identity

(b) $X' + XY = X' + Y$

Proof:

Collapse Theorem (see lecture notes)

X′ + XY	= (X′ + X).(X′ + Y)	Distributivity
	= 1.(X′ + Y)	Complementation
	= X′ + Y	Identity

Question 3. [8 marks]
Answer each question below. Each part is worth 2 marks. Show your work to earn partial marks if your answer is not completely correct.

A. Convert the decimal number **-374** to a signed-binary number using 2's complemention and a 16-bit representation.

```
Start by converting the absolute value:
374 / 2 = 187    rem  0   least significant (rightmost) bit
187 / 2 =  93    rem  1
 93 / 2 =  46    rem  1
 46 / 2 =  23    rem  0
 23 / 2 =  11    rem  1
 11 / 2 =   5    rem  1
  5 / 2 =   2    rem  1
  2 / 2 =   1    rem  0
  1 / 2 =   0    rem  1   most significant (leftmost) bit

Gathering bits yields the 12-bit result:   101110110

Now convert to 16-bit signed binary form.  First add seven
0-bits to the left side.

    0000000101110110
    1111111010001001   Complement
    1111111010001010   Add 1 - Final answer
```

B. Convert the octal number $\mathbf{525375_8}$ to a hexadecimal number.

```
Write octal digits as binary - groups of 3 bits per octal
digit
     = 101 010 101 011 111 101

Rearrange bits (from right) in groups of 4 bits
       = 0010 1010 1010 1111 1101

Convert each 4-bit group to a hex-digit (forget the left
0's)
       = 2AAFD
```

C. Convert the positive decimal real number **`21.6`** to an unsigned binary number.

Separate the number 21.6 into its integer and fractional parts and convert each separately. First, convert the integer:

```
21 / 2  =  10   rem  1
10 / 2  =   5   rem  0
 5 / 2  =   2   rem  1
 2 / 2  =   1   rem  0
 1 / 2  =   0   rem  1               → 10101
```

Next, convert the fraction:

```
 0.6 x 2  = 1.2 integer  1
 0.2 x 2  = 0.4 integer  0
 0.4 x 2  = 0.8 integer  0
 0.8 x 2  = 1.6 integer  1
 And this repeats.                  → 0.[1001]
 Where the bits within braces repeat.
```

D. State the <u>exact values</u> of the **largest positive** value and the **smallest negative** value that can be defined in an L-bit representation using signed-binary (ie. 2's complement) notation.

Largest positive value: $2^{L-1} - 1$

Smallest negative value: -2^{L-1}

Question 4. [6 marks]
State the six (6) fundamental logic gates (excluding the inverter) and, for each gate, provide a complete truth table that fully defines the logic of each gate. Each correctly identified gate and truth table is worth 1 mark.

AND

X/Y	0	1
0	0	0
1	0	1

OR

X/Y	0	1
0	0	1
1	1	1

NAND

X/Y	0	1
0	1	1
1	1	0

NOR

X/Y	0	1
0	1	0
1	0	0

XOR

X/Y	0	1
0	0	1
1	1	0

NXOR

X/Y	0	1
0	1	0
1	0	1

Question 5. [6 marks]
Answer all parts of this question. Each part is worth 2 marks.

Part A. Simplify the following Boolean expression using Boolean algebra.

A′B + ABC′ + ABC

ANSWER:
A′B + ABC′ + ABC = A′B + AB(C′+C)

= A′B + AB

= (A′+A)B

= B Final Answer

Part B. Simplify the following Boolean expression using Boolean algebra.

(BC′ + A′D)(AB′ + CD′)

ANSWER:
(BC′ + A′D)(AB′ + CD′)

= BC′AB′ + BC′CD′ + A′DAB′ + A′DCD′

= 0 + 0 + 0 + 0

= 0 Final Answer

Part C. Using De Morgan's theorem, show that:

(A + B)′(A′ + B′)′ = 0

ANSWER:
(A + B)′(A′ + B′)′

= A′B′AB = A′AB′B = 0.0 = 0 Final Answer

Question 6. [8 marks]

Part A. [4 marks]
Simplify the following Boolean SOP expression using the Karnaugh map technique. Express your answer in SOP form.

```
F(A,B,C) = ∑ m(3,5,6,7)
```

ANSWER:

Kmap	BC/00	01	11	10
0			1	
A/1		1	1	1

The resulting SOP form is:

F = AC + AB + BC

Part B. [4 marks]
Simplify the following Boolean SOP expression, with don't care conditions, using the Karnaugh map technique. Express your answer in POS form.

```
F(W,X,Y,Z) = ∑ m(2,5,7,8,9,12,15)

dc(W,X,Y,Z) = ∑ m(0,1,10,14)
```

ANSWER: (D is used for don't care values)

Kmap	YZ/00	01	11	10
WX/00	D	D	0	1
01	0	1	1	0
11	1	0	1	D
10	1	1	0	D

NOTE: Remember the D values can be chosen arbitrarily. In this example, there is no benefit gained by setting any D to 0. To form the POS expression, all D values are considered as 1's and only the 0-cells are encircled.

The resulting POS form is:

F = (W + X' + Z) (X + Y' + Z') (W' + X' + Y + Z')

Question 7. [5 marks]
Design a combinational circuit that detects an error in the representation of a decimal digit in BCD notation. In other words, obtain a logic diagram whose output is 1 when the inputs contain any one of the six unused bit combinations in the BCD code. Assume the input is a 4-bit BCD code called X (or the tuple { X_3 X_2 X_1 X_0 }) and the output bit is E (for Error bit).

ANSWER: The truth table below contains all six entries for which E=1. Note that these correspond to the unused bit combinations in the BCD code.

X_3	X_2	X_1	X_0	E
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

Hence, from the table we have the SOP expression:

$E = X_3 (X_2' X_1 X_0' + X_2' X_1 X_0 + X_2 X_1' X_0' + X_2 X_1' X_0 + X_2 X_1 X_0' + X_2 X_1 X_0)$

This can be simplified further, either algebraically, or using a K-map:

$\mathbf{E = X_3 X_2 + X_3 X_1}$

This page contains information provided freely for each student to use for the examination, if required. You may detach this page.

Boolean Postulates:

P0: Existence: There exist at least two elements x,y in B such that x ≠ y

P1: Closure: For every x,y in B there exist two combinational operators + and . where (x+y) is in B and (x.y) is in B

P2: Identity: There exist identity elements 0,1 in B relative to the operations + and ., such that for every x in B:

0+x = x+0 = x and 1.x = x.1 = x.

P3: Commutativity: The operations + and . are commutative for all x,y in B:

x+y = y+x and x.y = y.x

P4: Distributivity: Each operation + and . is distributive over the other; that is, for all x,y,z in B:

x.(y+z) = x.y+x.Z and x+(y.z) = (x+y).(x+z)

P5: Complementation: For every element x in B there exists an element ~x, called the complement of x, satisfying:

x+~x = 1 and x.~x = 0