60-265 Computer Architecture I: Digital Design Fall 2012

Exercise 1 – Numeracy

Review Chapter 1 of the textbook.

Question 1. [4 marks]

1. Express each value as a binary string of 0's and 1's.

   641_{10} = $1*512 + 1*128 + 1*1 = 1*2^9 + 1*2^7 + 1*2^0 = 1010000001_2$

   1327.1875_{10} **Separate integer and fraction parts:**

   Integer: Use successive division, accumulate remainders:
   $1327 = 2^{10} + 2^8 + 2^5 + 2^3 + 2^2 + 2^1 + 2^0$;

   Fraction: Use successive multiplication, accumulate integer parts:
   $.1875 = 1/8 + 1/16 = 2^{-3} + 2^{-4}$;

   Answer: 10100101111.0011_2

   'M' (printable character M - ASCII) **Hex 4D = 100 1101**

   'm' (printable character m - ASCII) **Hex 6D = 110 1101 Note the 1 bit difference with 'M'**

   ACK (communication signal - ASCII) **Hex 06 = 000 0110 Used to ACKnowledge a message**

   $2E4B_{16}$ **Express each hex digit using 4 bits, hence: 0010 1110 0100 1011 -> 0010111001001011**

2. Express the binary string **0101010100111101** in base-10, base-8 and base-16 representations.

   Base-16: Groups of 4 bits -> **0101 0101 0011 1101**, express hex digits -> **$553D_{16}$**

   Base-8: Groups of 3 bits -> **0 101 010 100 111 101**, express octal digits -> **52475_8**

   Base-10: Multiply each 1 by its corresponding power of 2, hence:

   $2^{14} + 2^{12} + 2^{10} + 2^8 + 2^5 + 2^4 + 2^3 + 2^2 + 2^0 =$
   $= 16384 + 4096 + 1024 + 256 + 32 + 16 + 8 + 4 + 1$
   $= 21821_{10}$

Question 2. [2 marks]

1. Express the number **-1_{10}** in 2's-complement integer form, assuming that the representation consists of (a) 4 bits, and (b) 8 bits.

ANSWER:
Consider a 2-bit representation of **1_{10}**, namely 01_2. The 1's complement is 10_2, and adding 1 yields the 2's complement, 11_2. Generalizing this gives a string of 1's for any size representation, hence:

4-bit: 1111_2 8-bit: 1111_2

2. Express the number **-4321_{10}** in 2's-complement integer form, assuming that the representation consists of 16 bits.

ANSWER:
Start with the absolute (ie. non-negative value), hence 4321, and generate a 16-bit binary string.

$4321 = 4096 + 128 + 64 + 32 + 1 = 2^{12} + 2^7 + 2^6 + 2^5 + 2^0 = 1000011100001$

Pad the left side with 0's to complete the 16-bit representation: 001000011100001
Next, complement the previous result to get: 110111100011110
Finally, add 1 to the previous result to get the 2's complement form of -4321: 110111100011111

3. Express the minimum (most negative) and maximum (most positive) values of a 16 bit 2's-complement integer representation. Provide both the binary strings and also their signed decimal (base-10, with sign +/-) equivalent values.

ANSWER:
The most negative number, the minimum value, for a 16-bit 2's complement integer representation is found by placing a 1 in the leftmost bit position, and 0's in all other positions, hence: 10000000 00000000. This number is just $-2^{16-1} = -2^{15} = -32{,}768$.

The most positive number, the maximum, starts with a 0 in the leftmost position, and has 1's in all other positions, hence: 01111111 1111111 = $2^{15} - 1 = 32{,}767$.

4. Express the number **-1234_{10}** in <u>both</u> 9's-complement <u>and</u> 10's-complement integer form.

ANSWER:
9's Complement form: 9999 – 1234 = 8765

10's Complement form: (9's Complement) + 1 = 8765 + 1 = 8766

Question 3. [1 mark]

1. Convert the number **$4CG4_{19}$** (base-19) to its equivalent value in base-10, and also in base-13. The digits in base-19 extend from 0 to 9, then use A, B, C, D, E, F, G, H and I.

ANSWER:
Convert the number to decimal, for convenience and because you have to achieve this anyway.

$\mathbf{4CG4_{19}} = 4x19^3 + 12x19^2 + 16x19^1 + 4x19^0 = 4x6859 + 12x361 + 16x19 + 4 = 32{,}076_{10}$

Now, take your previous result in base-10, and convert to base-13 using successive division and accumulation of remainders.

```
13 )  32076
        2467   5
         189  10
          14   7
           1   1
           0   1      ANSWER:  117A5(13)
```

Question 4. [1 marks]

1. Consider the array of asterisks below. Assume that each asterisk is actually a LED that shines when a voltage (value 1) is applied, but is off (value 0) when no electricity is applied. Using the basic arrangement shown, it is possible to light up LEDs such that all of the visual patterns for digits 0, 1, and so on up to 8, 9 are displayed. Design a bit-string consisting of 1's and 0's so that each position of a bit value corresponds to a specific asterisk in the LED pattern. Show for the decimal digits {0, 1, 3 and 9} what the accompanying bit patterns must be to produce the proper lighting up of LEDs.

```
******      ****
*          *    *
*          *    *
*****      ****
     *     *    *
     *     *    *
*****      ****
```

Figure. Example LED-on pattern for decimal digits 5 and 8.

ANSWER:
Begin by defining the bit sequence – since you need 42 bits to form a rectangular array, one for each LED lightbulb, you label these bits, say, as b0, b1, b2 … b41. Suppose we start at the upper left, top row with b0 controlling that LED, then b1 the LED to the next right and so on – bits b0, b1, b2, b3, b4 and b5 control the top row. Similarly, we proceed to the second row, third row and so on until you get to the bottom row, right hand side with b41. By assuming a particular LED pattern to give the visual appearance of the digits 0, … 9, you must control this by turning some LED`s ON (1) and the others OFF (0) in the bit string [b41,b40,…b2,b1,b0].

Alternative correct solutions can be generated, each depending on the labeling of the LED bits, but in all cases a 42-bit string is required.

Question 5. [2 marks]
Determine the base of the numbers in each case for the following operations to be correct (ie. for the integers in the bases shown).

1. 14/2 = 5
ANSWER: Solve by expanding in radix R, hence $14_R = R+4$, hence R+4=10 and R=6.

2. 24+17=40

ANSWER: Solve by expanding in radix R, hence $24_R + 17_R = 40_R$, hence 2R+4+R+7=4R, then 11=R.

FINAL NOTES:

Assigned Reading : Read Chapter 1 of the textbook, reviewing Gray codes, and excess-N representations of decimal (BCD) numbers. You may be tested on this material, but responsibility for the assigned reading and learning lies with each individual student.

Evaluation:

A. Students are evaluated on all stated requirements.
B. It is mandatory that students complete their own work and must be able to justify their answers when asked to do so by instructors and teaching staff.