60-265 Computer Architecture I: Digital Design Fall 2012

Exercise 3 – Karnaugh Maps

Question 1. SOP Karnaugh Maps [3 marks]

Karnaugh maps are often used to obtain simplified Sum of Product (SOP) expressions. Process each of the SOP expressions below, using Karnaugh maps. Each expression uses *canonical minterm* notation that may be used to fill in the K-map.

F(X,Y,Z) = SUM m(0,2,5,7)

The connected boxes are colour coded in the table below.

F(x,y,z)	00	01	11	10
0	1	0	0	1
1	0	1	1	0

Simplified expression: **F(X,Y,Z) = X'Z' + XZ** (also X xnor Z).

G(A,B,C,D) = SUM m(1,3,4,6,9,11,12,14)

The connected boxes are colour coded in the table below.

G(A,B,C,D)	00	01	11	10
00	0	1	1	0
01	1	0	0	1
11	1	0	0	1
10	0	1	1	0

Simplified expression: **G(A,B,C,D) = BC' + B'D**

Question 2. POS Karnaugh Maps [3 marks]

Karnaugh maps are also used to obtain simplified Product of Sums (POS) expressions. Process each of the POS expressions below, using Karnaugh maps. Each expression uses either *canonical minterm* or *canonical maxterm* notation that may be used to fill in the K-map.

H(X,Y,Z) = PROD M(0,2,3,4,6)

H(X,Y,Z)	00	01	11	10
0	0	1	1	0
1	1	1	1	0

$H(X,Y,Z) = (X+Z)(Y'+Z)$

L(A,B,C,D) = SUM m(4,6,7,15)

L(A,B,C,D)	00	01	11	10
00	0	0	0	0
01	1	0	1	1
11	0	0	1	0
10	0	0	0	0

$L(A,B,C,D) = (B)(C+D')(A'+D)$

Question 3. Don't Care Conditions in Karnaugh Maps [4 marks]

Simplify the Boolean function F together with the don't-care conditions dc in (1) SOP form and (2) POS form. Use the don't-care conditions to obtain expressions with the minimum number of literals.

F(W,X,Y,Z) = SUM m(0,1,2,3,7,8,10)

dc(W,X,Y,Z) = SUM m(5,6,11,15)

The K-map is provided below, first for the SOP case, then for POS. The resulting simplified expressions are provided below each K-map. Note that the POS expression has been expanded to demonstrate the equivalence of SOP to POS representations. In each K-map study which "don't care" entries (labeled D) are used and which are not. The SOP expression is not unique, but the choice indicated below utilizes the identification of *essential prime implicants*.

SOP::

F(W,X,Y,Z)	00	01	11	10
00	1	1	1	1
01	0	D	1	D
11	0	0	D	0
10	1	0	D	1

$F(W,X,Y,Z) = W'X' + X'Z' + W'Z$

POS::

F(W,X,Y,Z)	00	01	11	10
00	1	1	1	1
01	0	D	1	D
11	0	0	D	0
10	1	0	D	1

$F(W,X,Y,Z) = (X'+Z)(W'+Z') = W'X' + W'Z + X'Z'$

Additional Assigned Reading and Self-study Exercises:

Review and attempt all problems 3.1 to 3.30 at the end of Chapter 3 in the textbook. It is not required that students submit their work, nor will it be evaluated. However, examination questions may be based on these problems, so it is worthwhile to complete this work.

Evaluation:

A. All Laboratory Exercises must be completed and submitted for grading by the following Laboratory session, unless otherwise prescribed by the Instructor.
B. Students are evaluated on all stated requirements.
C. It is mandatory that students complete their own work and must be able to justify their answers when asked to do so by teaching staff.